Teaching the Design of Takagi-Sugeno Fuzzy Controllers for an Under-Graduate Course

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Abstract. Fuzzy control can be used to improve existing controller systems by adding an extra layer of intelligence to the current control method. This paper deals with a survey of Takagi-Sugeno fuzzy model and its controller. Two problems namely, inverted pendulum model and ball-and-beam model have been simulated and the results are shown. This survey is used to teach the design of Takagi-Sugeno fuzzy controllers for under-graduate courses.

1 Introduction

Substantial changes have undergone in the past years in the curricula for computer science and engineering [2]. Industry also demands the changes in the curricula by introducing new courses with hands on experience in writing softwares for real applications. The combination of hands-on experience and computer simulation with the more traditional, theoretical lecture material provides a well-rounded learning experience that better prepares the students for implementation in the real world [4].

The traditional method of introducing undergraduates to the areas that involve fuzzy logic and fuzzy controllers is through upper level elective courses [1]. The curricula for such an elective should be designed in such a way that math/theory side should not be sacrificed in favor of demos using high-level packages, but at the same time keeping in mind, that students cannot implement even a toy system if they must do it all from the scratch.

With these thoughts in mind, this paper focuses on teaching the design of the newest technology of Takagi-Sugeno fuzzy controllers for undergraduate courses with little in-depth treatise on the theory of fuzzy sets and fuzzy control.

The required background for the students includes knowledge in fuzzy logic and good programming skill which is essential to develop and test various real time applications. MATLAB [3, 6, 19, 20] software is preferred for programming, since it allows students to investigate the characteristics of the algorithm and

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Control, Virtual Instrumentation and Digital Systems.
Research in Computing Science 24, 2006, pp. 131-140

easily design their algorithm with its vast assortment of graphical and simulation functions.

This paper is organized as follows. In section 2 a survey of TS fuzzy model and its corresponding PDC controller design techniques are discussed. Section 3 discusses the application of TS fuzzy controller for inverted pendulum system and ball and beam system. Section 4 concludes the paper.

2 Takagi-Sugeno fuzzy model and its controller

Modeling techniques are believed to contain hidden intelligence that is able to solve a problem whenever the experimenter doesn't know what he is doing. A model is just a mapping from a given input to a given output. The essential element for the study of a nonlinear control problem is to get a tractable model of a dynamical system for use in control system design. The design model should be simple enough to work with, but must retain the essential features of the process.

Literature says that Takagi-Sugeno (TS) fuzzy model[21] along with Parallel Distributed Compensation (PDC) [23] scheme is best for control system design[3],[7]-[19]. This section deals with the survey of TS fuzzy model and its controller.

One can construct a Takagi-Sugeno fuzzy model if local description of the dynamical plant to be controlled, given by,

$$\dot{X} = f(X, u)$$

is available in terms of linear models

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, 2, \ldots, r$$

where the state vector $x(t) \in \Re^n$, the control input $u(t) \in \Re^m$, and the matrices A_i and B_i are of appropriate dimensions. The above information is then fussed with the available IF-THEN rules where the i^{th} rule can have the form

$$R_i: \text{ IF } x_1(t) \text{ is } F_1^i \text{ AND} \ldots \text{AND } x_n(t) \text{ is } F_n^i \text{ THEN } \dot{x}(t) = A_i x(t) + B_i u(t)$$

where F_j^i, $j = 1, 2, \ldots, n$, is the the j^{th} fuzzy set of the i^{th} rule. Let $\mu_j^i(x_j(t))$ be the membership function of the fuzzy set F_j^i and let

$$w_i(t) = \prod_{j=1}^{n} \mu_j^i(x_j(t)), \; i = 1, 2, \ldots r$$

where r is the number of IF-THEN rules. Then, given a pair (x(t), u(t)), the resulting fuzzy system model is inferred as the weighted average of the local models and has the form

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(t)(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} w_i(t)} = \sum_{i=1}^{r} \alpha_i(t)(A_i x(t) + B_i u(t)) \qquad (1)$$

where

$$\alpha_i(t) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)} \text{ for } i = 1, 2, \ldots, r$$

Note that $\alpha_i(t) \geq 0$ and $\sum_{i=1}^{r} \alpha_i(t) = 1$.

The open loop system corresponding to (1) is

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(t)(x) A_i x(t) \tag{2}$$

Theorem 1. *[22] The equilibrium of a fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix P such that*

$$A_i^T P + P A_i < 0 \quad i = 1, 2, \cdots, r \tag{3}$$

It is obvious that a necessary condition for the existence of a common symmetric positive definite P satisfying (3) is that each A_i be asymptotically stable; that is, the eigenvalues of each A_i be in the open left-hand complex plane.

Parallel Distributed Compensation (PDC) [23] can be utilized to design fuzzy controller rules. The idea is to design a compensator for each rule of the fuzzy model. The i^{th} rule of the controller is of the form

$$R_i: \text{ IF } x_1(t) \textit{ is } F_1^i \text{ AND} \ldots \text{AND } x_n(t) \textit{ is } F_n^i \text{ THEN } u(t) = -K_i x(t)$$

The overall fuzzy controller is obtained by fuzzy blending [23] of each individual linear controllers and is given by

$$u(t) = \frac{\sum_{j=1}^{r} -w_j(K_j x(t))}{\sum_{j=1}^{r} w_j} \tag{4}$$

which is nonlinear.

Substituting (4) into the fuzzy system (1), we obtain

$$\dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_j(x)\alpha_i(x)\left\{A_i - B_i K_j\right\} x \tag{5}$$

Theorem 2. *[24] The equilibrium of a fuzzy control system (5) [i.e., Fuzzy controller (4)+ Fuzzy Model (1)] is asymptotically stable in the large if there exists a common positive definite matrix P such that the following conditions are satisfied:*

$$\{A_i - B_i K_j\}^T P + P\{A_i - B_i K_j\} < 0 \qquad i, j = 1, 2, \cdots, r \tag{6}$$

It is obvious that a necessary condition for the existence of a common symmetric positive definite P satisfying (6) is that each $A_i - B_i K_j$ be asymptotically stable; that is, the eigenvalues of each $A_i - B_i K_j$ be in the open left-hand complex plane.

The system (5) can also be written as

$$\dot{x} = \sum_{i=1}^{r} \alpha_i(x)\alpha_i(x)\{A_i - B_iK_i\}x + 2\sum_{i<j} \alpha_i(x)\alpha_j(x)G_{ij}x \quad (7)$$

where

$$G_{ij} = \frac{\{A_i - B_iK_j\} + \{A_j - B_jK_i\}}{2} \quad i < j \quad (8)$$

Therefore the following sufficient condition is obtained.

Theorem 3. *[24] The equilibrium of a fuzzy control system (5) [i.e., Fuzzy controller (4)+ Fuzzy Model (1)] is asymptotically stable in the large if there exists a common positive definite matrix P such that the following two conditions are satisfied:*

$$\{A_i - B_iK_i\}^T P + P\{A_i - B_iK_i\} < 0 \quad (9)$$
$$i = 1, 2, \cdots, r$$
$$G_{ij}^T P + PG_{ij} < 0 \quad (10)$$
$$i < j \leq r$$

Pole placement method or linear quadratic control design method or Genetic algorithms or Linear Matrix Inequalities (LMI)[5, 22] can be used to find out the values of the Gains, K_is. Since the course if for undergraduate students, the gain values are obtained using linear quadratic method without going in-depth treatise on the theoretical part or stability issues.

3 Control Applications

3.1 Inverted Pendulum

A famous benchmark problem namely the inverted pendulum control problem is chosen for study. The non-linear and non-stable behavior of the inverted pendulum problem renders the use of conventional method very difficult.

The inverted pendulum system consists of a pole hinged on a cart as shown in Figure 1a. The control objective is to balance the inverted pendulum for the approximate range of vertical angle, namely $x_1 \in (-\pi/2, \pi/2)$. The equations of motion of the pendulum [23] are

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_2)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)}$$

where x_1 denotes the angle (in radians) of the pendulum from the vertical, and x_2 is the angular velocity, g is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, 2l is the length of the pendulum, u is the force applied to the cart and a =1/(m+M). In our simulation we used the values, g=9.8 m/s^2, m=2Kg, M=8Kg, a=0.1, 2l=1m, $\beta = \cos 88°$.

We approximate the system by the following two-rule Takagi Sugeno fuzzy model

$$R_1: \text{ If } x_1 \text{ is about } 0 \text{ then } \dot{x} = A_1x + B_1u$$
$$R_2: \text{ If } x_1 \text{ is about } \pm\pi/2 \text{ then } \dot{x} = A_2x + B_2u$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-aml} & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ \frac{-a}{4l/3-aml} \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-aml\beta^2)} & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ \frac{-a\beta}{4l/3-aml\beta^2} \end{bmatrix}$$

By utilizing the concept of Parallel Distributed Compensation (PDC), the following two rules are designed for the controller.

$$R_1: \text{ If } x_1 \text{ is about } 0 \text{ then } u_1 = K_1x$$
$$R_2: \text{ If } x_1 \text{ is about } \pm\pi/2 \text{ then } u_2 = K_2x$$

By choosing the eigen values [-2+2i, -2-2i], the controller gains K_1 and K_2 are obtained as [-143.3333 -22.6667] and [-3315.6 -764] respectively. Figure 1b shows the plot of θ starting from 16, 30, 79 and 87 degrees. Figure 1c and 1d shows the plot of $\dot{\theta}$ and u respectively.

3.2 Ball and Beam System

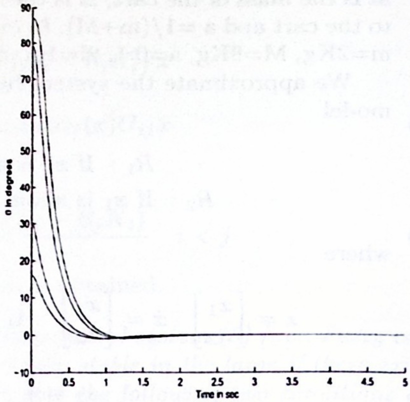
A ball is placed on a beam, where it is allowed to roll along the length of the beam. A motor at the center of the beam will apply torque to the beam. See Figure 2a. When the angle of the beam is changed from the vertical position due to the torque, gravity causes the ball to roll along the beam. A controller will be designed for this system so that the ball's position can be manipulated. We assume that the ball rolls without slipping and friction between the beam and ball is negligible.

The equation of motion for the ball is given by

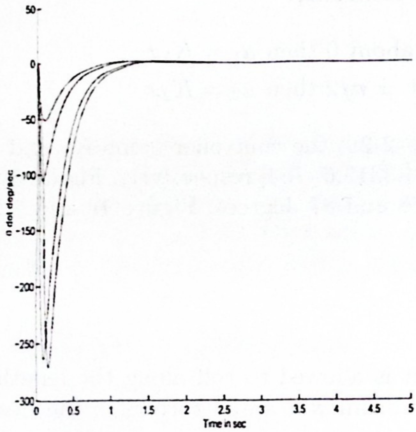
$$\ddot{r} = \frac{-mg\sin\alpha + mr\dot{\alpha}^2}{\frac{J}{R^2} + m} \tag{11}$$

where, m is the mass of the ball, r is the radius of the ball, l is the length of the beam, J is the ball's moment of inertia, r is the ball position coordinate, α is the beam angle coordinate. In our simulation the values used are m=0.11Kg, r=0.015m, g=9.8m/s^2, J=9.99×10^{6}kgm^2.

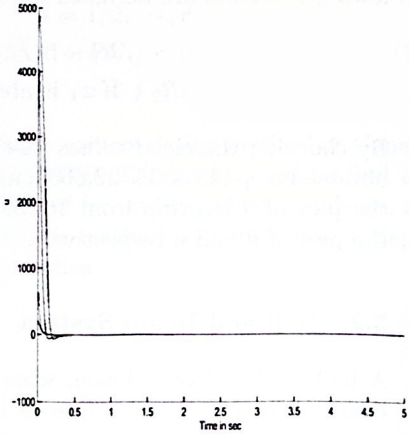
(a) Inverted pendulum



(b) Response of the θ



(c) Response of $d\theta/dt$



(d) Response of u

Fig. 1. Simulation results of Inverted Pendulum.

We approximate the system by the following 5-rule Takagi Sugeno fuzzy model. Table 1 shows the membership function labels used in the rules.

$$\text{If } \alpha \text{ is } Z \text{ Then } \dot{x} = A_1x + B_1u$$
$$\text{If } \alpha \text{ is PS or NS Then } \dot{x} = A_2x + B_2u$$
$$\text{If } \alpha \text{ is PM or NM Then } \dot{x} = A_3x + B_3u$$
$$\text{If } \alpha \text{ is PL or NL Then } \dot{x} = A_4x + B_4u$$

$$\text{If } \alpha \text{ is PVL or NVL Then } \dot{x} = A_5 x + B_5 u$$

$$\text{where } x = \begin{bmatrix} r \\ \dot{r} \\ \alpha \\ \dot{\alpha} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{r} \\ \ddot{r} \\ \dot{\alpha} \\ \ddot{\alpha} \end{bmatrix} \quad A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-mga_i}{\frac{J}{R^2}+m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

for $i = 1, 2, \ldots, 5$ and with $a_1 = 0$, $a_2 = 12\sin(15\pi/180)/\pi$, $a_3 = 3\pi$, $a_4 = 4/(\sqrt{2}\pi)$, $a_5 = 3\sqrt{3}/(2\pi)$

By utilizing the concept of Parallel Distributed Compensation (PDC), the following 5 rules are designed for the controller.

$$\text{If } \alpha \text{ is } Z \text{ Then } u = k_1 x$$
$$\text{If } \alpha \text{ is PS or NS Then } u = k_2 x$$
$$\text{If } \alpha \text{ is PM or NM Then } u = k_3 x$$
$$\text{If } \alpha \text{ is PL or NL Then } u = k_4 x$$
$$\text{If } \alpha \text{ is PVL or NVL Then } u = k_5 x$$

By choosing the eigen values [-2+2i, -2-2i, -0.5, -1], the controller gains are obtained as

$$K_1 = [0.5714 \ \ 2.0000 \ \ 14.5 \ \ 5.5] \quad K_2 = [0.5780 \ \ 2.0230 \ \ 14.5 \ \ 5.5]$$
$$K_3 = [0.5984 \ \ 2.0944 \ \ 14.5 \ \ 5.5] \quad K_4 = [0.6347 \ \ 2.2214 \ \ 14.5 \ \ 5.5]$$
$$\text{and} \quad K_5 = [0.6910 \ \ 2.4184 \ \ 14.5 \ \ 5.5]$$

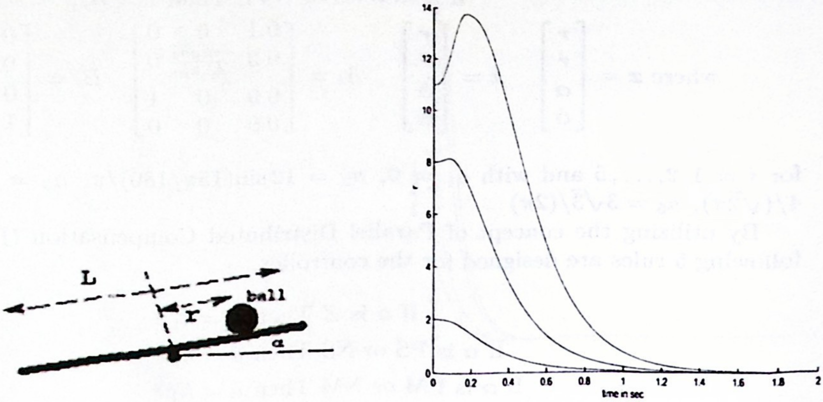
Figure 2b shows the plot of r starting from various positions. Figure 3a, 3b, 3c, and 3d shows the plot of $\dot{r}$, α, $\dot{\alpha}$ and u respectively.

Table 1. Membership function labels for α

Label	Left	Center	Right	Label	Left	Center	Right
NVL	-75	-60	-45	NL	-60	-45	-30
NM	-45	-30	-15	NS	-30	-15	0
Z	-15	0	15	PS	0	15	30
PM	15	30	45	PL	30	45	60
PVL	45	60	75				

4 Conclusion

This paper deals with the survey of Takagi-Sugeno fuzzy model and its controller. Two problems namely, inverted pendulum model and ball-and-beam model have been simulated and the results are shown. This survey and simulations using



(a) Ball and Beam system (b) Response of r

Fig. 2. Ball and Beam system and response of r

Matlab are used to teach the design of Takagi-Sugeno fuzzy controllers for undergraduate courses.

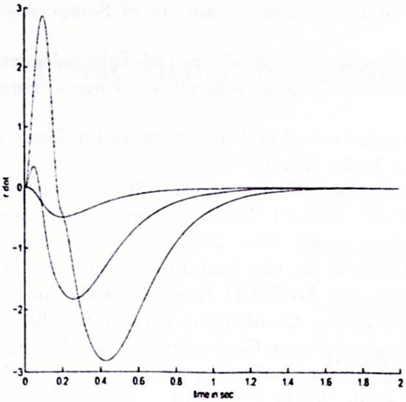
Implementations using fuzzy logic controllers can work well without having to construct any mathematical model of the process or plant. Fuzzy logic control yields results superior to those using conventional control algorithms and their applications can also lead to reduced development cost. In many cases, fuzzy control can be used to improve existing controller systems by adding an extra layer of intelligence to the current control method.

5 Acknowledgement

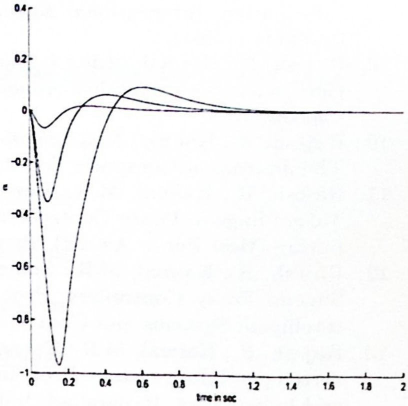
The authors would like to thank the reviewers for their valuable comments for improving the quality of the paper. The authors are thankful to all the staffs of the Department of Computer Science and Engineering, Bharathiar University, India.

References

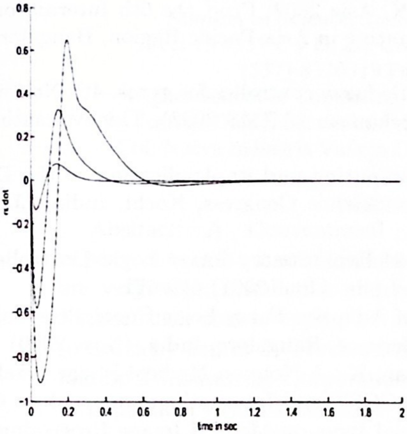
1. Bowyer, K., Stockman, G., Stark, L.: Themes for improved teaching of image computation. IEEE Trans. Education 43(2) (May 2000) 221-223.
2. Jurado, F., Castro, M., Carpio, J.: Experiences with fuzzy logic and neural networks in a control course. IEEE Trans. Education 45(2) (May 2002) 161-167.
3. Kaimal, M.R., Rajesh, R., Mohanlal, P.P.: An Optimal Rule Generation Scheme using Genetic Algorithm for the Design of a Fuzzy Logic Controller. PARITANTRA, Journal of System Society of India 8(1) (Aug. 2003) 20 - 25.



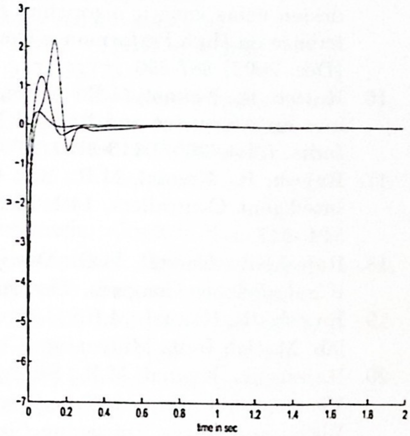
(a) Response of $\dot{r}$



(b) Response of α



(c) Response of $\dot{\alpha}$



(d) Response of u

Fig. 3. Simulation results of Ball and Beam system

4. Klee, H., Dumas, J.: Theory, Simulation, Experimentation-An integrated apprach to teaching digital control systems. IEEE Trans. Education 37(1) (Feb 1994) 57-62.
5. Lam, H.K., Leung, F.H.F., Tam, P.K.S.: A linear matrix inequality approach for the control of uncertain fuzzy systems. IEEE Cont. Sys. Mag. (Aug. 2002) 20-25.
6. Matlab Version 6.0 User Manual, MathWorks, Inc. Natick, MA, 2000.
7. Rajesh, R., Kaimal, M.R.: T-S Fuzzy Model with Nonlinear Consequence and PDC Controller for a Class of Nonlinear Control Systems. Applied Soft Computing Journal - in press (avilable online).

8. Rajesh, R., Kaimal, M.R.: Variable gain Takagi-Sugeno fuzzy logic controllers. Informatica, International Journal of Lithuanian Academy of Sciences - in press (avilable online).
9. Rajesh, R., Kaimal, M.R.: A Novel Method for the Design of Takagi-Sugeno Fuzzy Controllers with Stability Analysis using Genetic Algorithm. Engineering Letters - accepted
10. Rajesh, R., Kaimal, M.R.: Semi-circular membership functions for fuzzy systems. The Journal of Computer Society of India (2003) 40-47.
11. Rajesh, R., Kaimal, M.R.: Bounds of the Solution for the Stability Equations of Takagi-Sugeno Fuzzy Controllers. Proc. Second World Congress on Lateral Computing (Best Paper Award), Bangalore, India, Dec. 2005.
12. Rajesh, R., Kaimal, M.R.: Some Remarks on the Stability Conditions of Takagi-Sugeno Fuzzy Controllers. Proc. The 8th IASTED International Conference on Intelligent Systems and Control (ISC 2005), Cambridge, USA, Nov. 2005.
13. Rajesh, R., Kaimal, M.R.: Takagi-Sugeno Fuzzy Controller for six dimensional inverted pendulum system. Proc. International Conference on Systemics, Cybernetics and Informatics, Hyderabad, India, (Feb. 2004) 278 - 283.
14. Rajesh, R., Kaimal, M.R.: Modeling Inverted Pendulum - A Novel Approach. Proc. International Fuzzy Systems Associations World Congress, Turkey, 2003.
15. Rajesh, R., Kaimal, M.R.: Fuzzy logic controllers with guaranteed stability - A design using genetic algorithm. HPC Asia 2002, Proc. the 6th International Conference on High Performance Computing in Asia-Pacific Region, Bangalore, India, (Dec. 2002) 387-390.
16. Rajesh, R., Kaimal, M.R.: Automatic fuzzy controller for gyros. 4th National Seminar on Aerospace and Related Mechanism (ARMS 2002), Thiruvananthapuram, India, (Nov. 2002) 113-118.
17. Rajesh, R., Kaimal, M.R.: Soft Computing and its Application to the Design of Intelligent Controllers. 14th Kerala Science Congress, Kochi, India, (Jan. 2002) 514 -517.
18. Rajesh, R., Kaimal, M.R.: Design of Evolutionary Fuzzy Logic Controllers. 13th Kerala Science Congress, Thrissur, India, (Jan. 2001) 668-671.
19. Rajesh, R., Kaimal, M.R.: Design of Adaptive Fuzzy Logic Controllers Using Matlab. Matlab India Millennium Conference, Bangalore, India, (Nov. 2000) 167-180.
20. Rajesh, R., Kaimal, M.R., Srinivasan, K.: A Note on Medical Image Analysis and Visualization using Matlab. Proc. ICGST and International Journal on Graphics Vision and Image Processing (Special Issue on Medical Image Processing) (2006) 352-355.
21. Takagi, T., and Sugeno, M.: Fuzzy identification of systems and its applications to modleing and control. IEEE Trans. Syst. Man and Cyber. 15 (1985) 116-132.
22. Tanaka, K. and Sugeno, M.: Stability analysis and design of fuzzy control systems. Fuzzy Sets and Systems 45(2) (1992) 135-156.
23. Wang, H.O., Tanaka, K., Griffin, M.: Parallel distributed compensation of nonlinear systems by Takagi-Sugeno Fuzzy Model. IEEE .... (1995) 531-538.
24. Wang, H.O., Tanaka, K., Griffin, M.: An approach to fuzzy control of nonlinear systems - stability and design issues. IEEE Trans. on fuzzy systems (1996) 14-23.